Adrian Rezus

Lambda-calculus, Type Theories and Proof Theory, A Selective Bibliography 1981-2011 [*]

Rezus, A. (1981). **Lambda-conversion and Logic**, Elinkwijk BV, Utrecht 1981. (PhD Diss., University of Utrecht, 1981, under Dirk van Dalen and Henk Barendregt.)

Rezus, A. (1981a) "Analytical Indices and Bibliography of: H. P. Barendregt **The Lambda-Calculus, Its Syntax and Semantics**", North Holland, Amsterdam 1981 (first edition).

Rezus, A. (1982). "On a theorem of Tarski". *Libertas Math.*, 2:63-97.

[**MR 84b: 03019**] The theorem of the title was asserted as Theorem 8 in a paper by J. Lukasiewicz and A. Tarski [see Tarski, *Logic, semantics and metamathematics*, English translation, Clarendon Press, Oxford, 1956; **MR 17**, 1171]. It states: *Every finitely axiomatizable system of propositional calculus with the rules of substitution and detachment in which the two formulas CpCqp and CpCqCCpCqrr are provable can be axiomatized by a single formula.* According to the author, no proof of Tarski's theorem had been published prior to the present paper. The proof uses the formulas-as-types idea of H. B. Curry [Curry and R. Feys, *Combinatory logic*, Vol I, Theorem 9E, North-Holland, Amsterdam, 1958; **MR 20** #817] and the condensed detachment operator of C. A. Meredith [see, e.g., Meredith and A. N. Prior, *Notre Dame J. Formal Logic 4* (1963), 171-187; **MR 30** #1033]. Specifically, if α and β are formulas of propositional calculus, and there is a substitution yielding α' and β' where α' is Cβ'γ, then the value of the metaformula (αβ) is the most general form of any such, whence we write (αβ) = γ. The metaformulas constructed from propositional calculus formulas as atoms by the operation of application thus en code proofs of the formulas which are their values. For example, let **C** and **K** abbreviate CCpCqrCqCpr and CpCqp, respectively. Then for any α, (**CK**)α = Cpp, and, whenever α, β, and γ have the requisite forms to give values to the metaformulas, ((**C**α)β)γ = (αγ)β. These examples suggest an analogy with the theory of combinators, and in fact the equality of metaformulas is largely analogous to combinatory equality, which the author exploits to obtain a proof of Tarski's theorem. Noting that CpCqp is not valid in systems of relevant logic, the author modifies and refines his results to obtain single axioms for the purely implicational fragments of various relevant logics. He states as an open problem the existence of single axioms for relevant logics which include conjunction or disjunction. In an appendix he shows that every closed λ-calculus term is equal to an applicative combination of the single term λxyz.y(λu.z)(xz). (Reviewed by B. Lercher.)

Rezus, A. (1982). **A Bibliography of Lambda-calculi, Combinatory Logics and Related Topics**, Mathematisch Centrum, Amsterdam 1982. (With a preface by Henk Barendregt.)

[**MR 83: 03010**] From the preface: "The items in this bibliography fall into the following categories: (1) the pure theory (syntax and semantics); (2) the theory related to foundations (of logic and mathematics); (3) applications (recursion theory, computer science, proof theory, category theory)."

Rezus, A. (1983). **Abstract Automath**, Mathematisch Centrum, Amsterdam 1983 [*Mathematical Centre Tracts 160*] (Revision of a Technical Report of the Mathematics Department, University of Technology Eindhoven, 1982. Also available as an electronic publication from "The Automath Archive", Technische Universiteit, Eindhoven: http://www.win.tue.nl/automath/).

[**MR 84j: 03030**] It is an old idea to represent mathematics in a pure formal system. Such attempts are connected with names like Leibniz, Peano, Whitehead and Russell, Hilbert and others. The formal language required for this purpose has to reflect many aspects: the theorems, the proofs, the ideas of the proofs, the formal and informal background, etc. All historical approaches have failed insofar as they did not represent a completely formalized substantial part of mathematics without any gaps. In the mid- sixties N. G. de Bruijn initiated a far-reaching enterprise for the formalization of mathematics: AUTOMATH. In the meantime the project has been continuously developed and by now quite a number of AUTOMATH-languages exist for different purposes. The idea of this book is to represent the syntax and structure of some of these languages in some detail. The main parts are: Well-formedness, correctness, and its main aspects ("reference order", "complexity", "economy of means"). From the intention it is quite clear that the structure of such a "generalized typed lambda-calculus" is somewhat complicated. The reader is carefully guided and often motivated. Nevertheless, in order to get a good feeling for the system one will probably actually have to work with it and to use it in practice. (Reviewed by M. M. Richter.)

Barendregt, H. and Rezus, A. (1983). "Semantics for classical Automath and related systems". *Inform. and Control*, 59 (1-3):127-147. (Work supported in part by the University of Technology, Eindhoven.)

[**MR 86g: 03100**] The task of the AUTOMATH project is the formalization of (present day) mathematics so that proofs can be formally checked (e.g., by a computer). Although the languages of the AUTOMATH family are in some sense essentially typed lambda-calculi their structure is quite complicated; the publications in this area in general deal with the syntactic aspects. In this paper a formal semantics for AUTOMATH is provided. The domains of interpretation are models of the type-free lambda-calculus; hence the type distinctions of AUTOMATH have to be translated into a type-free setting. This leads to a notion of truth-evaluation similar to the one in first-order or second-order logic, and it is proved that the syntactic consequence operator is correct with respect to this semantics. (Reviewed by M. M. Richter.)

Rezus, A. (1986). "Semantics of constructive type theory", *Libertas Math.*, 6:1-82. (Revised as: Technical Report 2/1987 [n.s.], Department of Computer Science, University of Nijmegen [April] 1987. Work supported by NWO [The Hague], the Dutch National Science Foundation.)

[**MR 88a: 03034**] The author presents an interpretation of P. Martin-Löf's type theory CST [*Logic, methodology, and philosophy of science*, VI (Hannover, 1979), 153-175, North-Holland, Amsterdam, 1982; **MR 85d**:03112] with one universe and without identity types. In the author's approach, following D. S. Scott [*SIAM J. Comput.* 5 (1976), no. 3, 522-587; **MR 55** # 10262], types are translated into objects in a model of the type-free lambda calculus. The goal is to show that at least a subsystem of constructive type theory is intelligible from a classical non-constructivistic viewpoint. A second goal is to provide an abstract setting for the design of a large class of typed programming languages. The author's presentation includes an extensive discussion of related work on models of constructive type theory. (Reviewed by Henry Africk.)

Rezus, A. (1986a) **Impredicative Type Theories (Girard-Reynolds Typed Lambda-Calculi)**, Technical Report 85-1986, Department of Computer Science, University of Nijmegen 1986. (Revised Lectures Notes, Nijmegen 1985-1986. Work supported by NWO.)

Rezus, A. (1990) **Classical Proofs, Lambda-calculus Methods in Elementary Proof Theory**, Nijmegen 5.VII.1990. (Unpublished monograph draft. Work partly supported by NWO.)

Rezus, A. (1991) **Beyond BHK**, Nijmegen 1991, revised 1993. (Extended abstract in: Henk Barendregt, Marc Bezem, and Jan Willem Klop (eds.), **Dirk van Dalen Festschrift**, University of Utrecht 1993, pp. 114-120 [*Quaestiones Infinitae 5, Publications of the Department of Philosophy, Utrecht University*], also available as an electronic publication from www.equivalences.org.)

Rezus, A. (2010) "Tarski's Claim, thirty years later", Preprint Nijmegen, [October 1] 2010. (Unpublished; available as an electronic publication from www.equivalences.org.)

Rezus, A. (201+) **Witness Structures** (Monograph announced as of 1993; includes a revision of Rezus (1990); to appear.)

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[*] The reviews published in **Mathematical Reviews** have been excerpted from: Inge Bethke, **Annotated Bibliography of Lambda-Calculi, Combinatory Logics and Type Theory**, Preprint, University of Amsterdam [Amsterdam, October 29, 1999, 597 pp.]. Revised as: Henk Barendregt, Inge Bethke, and Silvia Ghilezan, **Lambda Calculi and Type Theory, 20-th Century and Beyond**, **A Sorted Bibliography Based on *Mathematical Reviews***, Preprint, Radboud University, Nijmegen [6.1.2004, 591 pp.]. Other papers (including several unpublished items, not mentioned in the above) are reviewed, discussed and / or referred to by title in **Zentralblatt für Mathematik und ihre Grenzgebiete**, **The Journal of Symbolic Logic**, as well as in a number of occasional surveys and monographs by other people, as, for instance, Giuseppe Longo, "*The Lambda-Calculus: connections to higher type recursion theory, proof-theory, category theory*", Lecture held at the Conference "Church's Thesis After Fifty Years" (Zeist, NL), June 1986 (revised as "*On Church's formal theory of functions and functionals*", in: **Annals of Pure and Applied Logic**, 40:93-133, 1988), A. S. Troelstra and Dirk van Dalen, **Constructivism in Mathematics**, Volumes 1-2, North Holland, Amsterdam 1988, Alan Ross Anderson, Nuel D. Belnap Jr., J. Michael Dunn et al., **Entailment, A Logic of Relevance and Necessity,** Volume 2, Princeton University Press, Princeton NJ 1992, J. Roger Hindley, **Basic Type Theory**, Cambridge University Press, Cambridge UK 1997, Bart Jacobs, **Categorical Logic and Type Theory**, Elsevier Science, Amsterdam [etc.] 1999, Morten Heine Sørensen, and Paweł Urzyczyn, **Lectures on the Curry-Howard Isomorphism**, North Holland / Elsevier, Amsterdam [etc.] 2006., etc. For historical and bibliographical details, see also Felice Cardone, and J. Roger Hindley, **History of Lambda-calculus and Combinatory Logic**, Swansea University Mathematics Department Research Report, MRRS-05-06, 2006 (revised as: "*Lambda Calculus and Combinators in the 20th century*", in: Dov M. Gabbay and John Woods (eds.) **Handbook of the History of Logic,** Volume 5: **Logic from Russell to Church**, North Holland / Elsevier, Amsterdam [etc.] 2009, pp. 722-817).